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Heat transfer characteristics of a rectangular natural circulation loop containing water near its density extreme

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Abstract—This article presents results of a theoretical and experimental study concerned with steady-state characteristics of a rectangular natural circulation loop with vertical heat transfer sections containing water near its density extreme. A one-dimensional model has been developed to simulate heat transfer behavior of the rectangular loop, with particular attention to unravel effect of density inversion of water near 4°C. In parallel to the theoretical simulations, supporting experiments have been undertaken to obtain data to verify the model. Fairly good agreement was found between the predicted and measured temperature distribution along the loop. Influence of density inversion effect has been clearly demonstrated to be substantially significant and needs to be taken into account when the operating temperature of the natural circulation loop filled with water encompassing its density-extreme temperature. © 1997 Elsevier Science Ltd.

INTRODUCTION

Natural circulation loops or thermosyphons, in which the fluid flow is induced by buoyancy force due to an arrangement of a heat source with heat sinks located at some height above the heat source, have received great research interest due to its wide range of engineering applications, such as nuclear reactor emergency cooling systems, solar heating and cooling systems, geothermal power generation, waste heat recovery system and so on. Extensive reviews of the state of the art related to natural circulation loop and its applications are available in [1–4].

Natural circulation in a single phase loop of various configurations and operating conditions has been the topic of extensive research in the existing literature, with the main impetus being the fundamental study of simple systems featuring typical nonlinear convective effects. Fluid flow and temperature field inside the natural circulation closed loop of different configurations and applications are mostly modeled utilizing one-dimensional (1-D) formulations incorporating with empirical expressions to account for some physical effects relevant to the buoyancy-driven heat and fluid transport process in the closed loop, as exemplified in Refs. [5–7]. Experiments have also been undertaken to verify the corresponding one-dimensional model predictions. The 1-D model has succeeded in describing some of the qualitative features of the natural circulation closed loop performance.

From the existing literature concerning natural circulation closed loop using water as the heat transfer medium, it can be noticed that the operating temperature ranges considered were usually far above that encompassing the density inversion phenomenon of water about 4°C. In the application of ice storage system incorporating with loop thermosyphon, the cooler section of the loop may attain a temperature low enough around 4°C and as a result, the density inversion effect may play a significant role in the heat transfer characteristics of the closed loop. Effects of the density inversion have been demonstrated on natural convection heat transfer in enclosures of different configurations, as revealed in the representative references [8–10]. To our best knowledge, there is no previous effort to examine the influence of density inversion on thermal performance of a natural circulation loop except for [11], in which the density inversion influence on the onset and steady-state flow in a parallel loop thermosyphon was analyzed. Accordingly, the present study aims to investigate the steady-state heat transfer characteristics of a rectangular natural circulation loop filled with water encompassing its density extreme to unveil the density inversion effect. Specifically, we extend the conventional 1-D model to account for the nonlinear density relation about 4°C. In parallel to the theoretical analysis of the circulation loop under consideration, experiments were undertaken to obtain data needed to verify the model predictions.

EXPERIMENTS

Shown schematically in Fig. 1 is the single-phase natural circulation loop used in the experiments. The

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NOMENCLATURE

A	cross-sectional area of the pipe, $\pi D^2/4$	s	axial coordinate around the loop
b	exponent of density equation	T	cross-sectional averaged fluid temperature
c_p	specific heat of the fluid at constant pressure	$\bar{T}_{cw}$	length-averaged wall temperature of the cooler section
D	internal diameter of the pipe	$\bar{T}_h$	length-averaged fluid temperature over the heater section
f	friction factor	$\bar{T}_{hw}$	length-averaged wall temperature of the heater section
g	gravitational acceleration	ΔT	temperature difference
h	heat transfer coefficient	V	cross-sectional averaged velocity of fluid
H	height of the rectangular loop	W	width of the rectangular loop
k	thermal conductivity	α	thermal diffusivity of fluid
L_t	total length of the closed loop, $2(H+W)$	β	thermal volumetric expansion coefficient
L_c	length of cooler section	ν	kinematic viscosity
L_{eff}	effective total length of the closed loop	ρ	density
L_h	length of heater section	τ_w	wall shear stress at the pipe.
Nu	Nusselt number, $q_h/\pi k L_h(\bar{T}_{hw}-\bar{T}_{cw})$		
p	cross-sectional averaged pressure of fluid		
P	perimeter of the pipe, πD		
Pr	Prandtl number, ν/α		
q''	heat flux at the pipe surface	Subscripts	
q	total transfer rate	c	referred to the cooled section
Ra	Rayleigh number, $g\beta(\bar{T}_{hw}-\bar{T}_{cw}D^3/\nu\alpha$	h	referred to the heated section
Re	Reynolds number, VD/ν	m	maximum density
rsp	coefficient of density equation	w	referred to pipe wall.

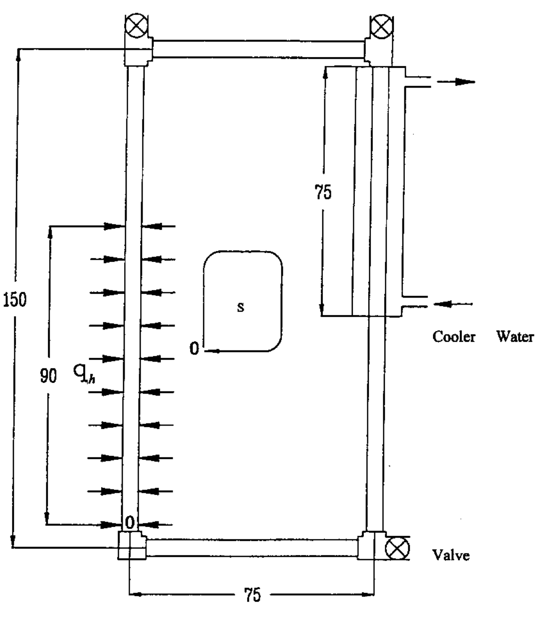

Fig. 1. Schematic diagram of the experimental natural circulation loop.

rectangular loop is composed of tubing of 1.6 cm inner diameter and is insulated with 6 cm of insulation tubing. The two vertical legs of the loop are constructed from circular stainless steel tubing while the two horizontal legs are the acrylic tubes. The heater section of length 90 cm was wrapped around by an electrically insulated heating tape and heated electrically with a D.C. power supply. Meanwhile, the cooler section of length 75 cm was cooled by a double-pipe heat exchanger connected to a constant temperature bath (Landau model RC-20), in which a mixture of water and ethanol was used as the heat exchange medium.

The loop was installed with a total of 32 copper–constantan thermocouples to monitor temperature of water inside the loop as well as of the tube wall at various locations as indicated in Fig. 2. Each thermocouple measuring the water temperature was positioned approximately at the cross-sectional center of the loop. In addition, five thermocouples were placed along the outside wall of the electrically heated section to determine the heat loss from the loop. The ambient temperature in the laboratory was monitored using another thermocouple. A data logger (Yokogawa, Model 3834) was used to record the temperatures. The uncertainty in measuring the temperature with the type T thermocouples and recording them with the data logger was estimated to be $\pm 0.3°C$. The experimental uncertainty associated with determining the heat input to the loop is estimated, according to [12], to be within 5%.

The experimental procedure was as follows. The

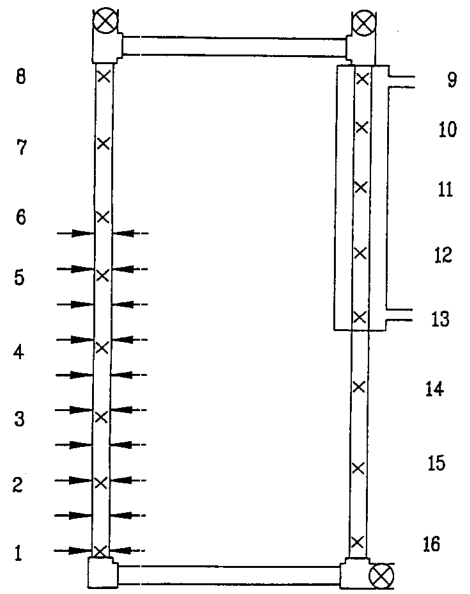

Fig. 2. Location of thermocouples in the loop.

loop was filled with distilled water. The constant temperature bath connected to the cooling jacket was first set to establish internally the temperature at a predetermined value. The heat exchange fluid of constant temperature bath was then switched on to circulate through the cooler jacket of the loop; meanwhile, the heating tape through the D.C. power supply was turned on to supply heat input at the heater section. Readings of the thermocouples were recorded until steady state was reached. After a period of approximately 2 hours, the steady state condition was established in the loop.

ANALYSIS

A 1-D theoretical analysis has been undertaken to predict the steady state heat transfer characteristics of a single-phase rectangular natural circulation loop mimicking the experiments conducted in the present study. In the model, the following assumptions were adopted. The internal diameter of the loop pipe is much smaller than its length. The buoyancy-driven flow of water in the loop is steady and laminar adhering to the Boussinesq approximation. The axial conduction heat transfer along the pipe wall and in the water is negligible in comparison with convection. The flow is assumed fully developed throughout the loop. Application of the conservation of mass, momentum, and energy for 1-D (cross-sectional averaged) buoyancy-driven flow about the loop gives the following set of governing differential equations.

$$\frac{\mathrm{d}V}{\mathrm{d}s} = 0 \tag{1}$$

$$\frac{\mathrm{d}p}{\mathrm{d}s} = -\frac{\tau_w P}{A} - \xi \rho g \tag{2}$$

$$\rho c_p V A \frac{\mathrm{d}T}{\mathrm{d}s} = q'' P \tag{3}$$

where $\xi = 1$ for the heater leg, $\xi = -1$ for the cooler leg and $\xi = 0$ for the horizontal legs of the rectangular loop. From equation (1) it is evident that the cross-sectional averaged flow velocity in the loop is constant. Moreover, in equation (3) $q'' = q''_h$ at the heater section, $q'' = -h_c(T - \bar{T}_{cw})$ at the cooler section and $q'' = 0$ otherwise. To model the density inversion phenomenon of water near 4°C, a density-temperature relation proposed by Gebhart and Mollendorf [13] was adopted.

$$\rho = \rho_m(1 - rsp|T - T_m|^b) \tag{4}$$

where $\rho_m = 999.972$ kg m^{-3}, $rsp = 9.297173 \times 10^{-6}$ (°C)$^{-b}$, $T_m = 4.029325$°C and $b = 1.894816$; in addition to the conventional linear density–temperature relation.

Integration of the momentum equation (2) along the loop yields

$$\frac{2V^2 f L_{\mathrm{eff}}}{D} = g \cdot rsp\left[\int_0^H |T - T_m|^b \mathrm{d}s - \int_{H+W}^{2H+W} |T - T_m|^b \mathrm{d}s\right] \tag{5}$$

where f represents the Fanning friction factor that $\tau_w = f(\rho V^2/2)$. It should be noted that an "effective" length, L_{eff} instead of the true total length, L_t, of the loop is used for the viscous friction term in equation (5) to account for both the form and frictional losses [7]. Furthermore, the friction factor f for the fully developed pipe flow is set equal to $16/Re$, where $Re = VD/\nu$. As a result, equation (5) becomes

$$V = \frac{1}{32}\frac{g \cdot rsp D^2}{\nu L_{\mathrm{eff}}}\left[\int_0^H |T - T_m|^b \mathrm{d}s - \int_{H+W}^{2H+W} |T - T_m|^b \mathrm{d}s\right] \tag{6}$$

For the condition of neglecting the density inversion effect of cold water, namely assuming a linear density–temperature relation, the momentum equation takes the form of

$$V = \frac{1}{32}\frac{g\beta D^2}{\nu L_{\mathrm{eff}}}\left[\int_0^H T\,\mathrm{d}s - \int_{H+W}^{2H+W} T\,\mathrm{d}s\right] \tag{7}$$

Next, the energy equation, equation (3), can be integrated along the loop to obtain the cross-sectional averaged fluid temperature distribution as follows: for $0 \leqslant s \leqslant L_h$ (heater section),

$$T(s) = T_0 + \frac{4q''_h s}{\rho c_p D V}. \tag{8a}$$

$L_h \leqslant s \leqslant L_t/2$

$$T(s) = T_1 = T(s = L_h) = T(s = L_t/2) = T_2. \quad \text{(8b)}$$

$L_t/2 \leqslant s \leqslant L_t/2 + L_c$ (the cooler section)

$$T(s) = \bar{T}_{cw} + (T_2 - \bar{T}_{cw}) \exp\left[-\frac{4h_c}{\rho c_p D V}(s - L_t/2) \right]. \quad \text{(8c)}$$

$L_t/2 + L_c \leqslant s \leqslant L_t$

$$T(s) = T_3 = T(s = L_t/2 + L_c). \quad \text{(8d)}$$

In equation (8c) the length-averaged wall temperature of the cooler section $\bar{T}_{cw}$ was based on the measured result of the corresponding experiments; while the heat transfer coefficient h_c was evaluated by the relation, according to $q_h''PL_h = q_c''PL_c$ under the steady state condition, $h_c L_c\ (\bar{T} - \bar{T}_{cw}) = q_h'' L_h$. Further it can be noticed from equations (8b) and (8d) that a uniform temperature distribution arises over the two adiabatic sections, leading to $T_0 = T_3$ and $T_1 = T_2$. It follows that under the steady-state condition the linear fluid temperature-rise over the heater section ΔT_h ($= T_1 - T_0 = 4q_h'' L_h/\rho c_p D V$) equals to the exponential fluid temperature-drop over the cooler section ΔT_c ($= T_2 - T_3$).

An iterative procedure was applied to equation (6) or (7) with equation (8) for the flow velocity and temperature distribution of the circulating fluid as well as the heat transfer coefficient h_c. The calculation started with evaluating the heat transfer coefficient based on an initial guess for the temperature distribution of fluid. Meanwhile, a new circulating flow velocity was determined from equation (6) or (7), depending on whether the density inversion effect was taken into account or not. Then, new fluid temperature distribution along the loop was updated from equation (8) based on the newly calculated values of flow velocity and heat transfer coefficient. The foregoing procedure was repeated until convergence was found for the temperature distribution and velocity of circulating fluid as well as the heat transfer coefficient.

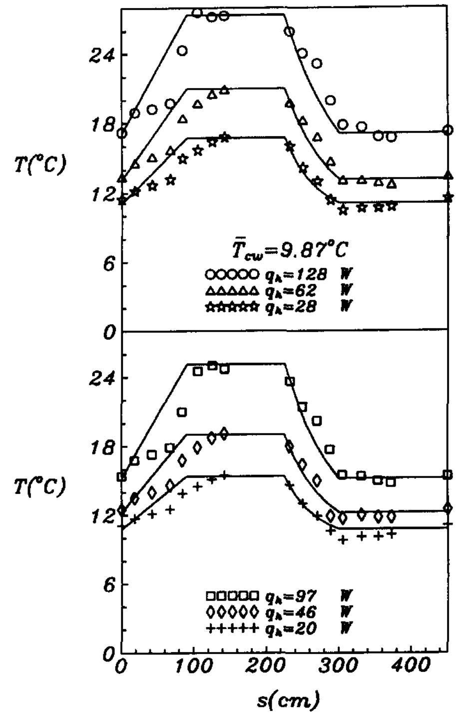

Fig. 3. Comparison of predicted and measured temperature distribution at various values of heat input with $\bar{T}_{cw} = 9.87°\mathrm{C}$.

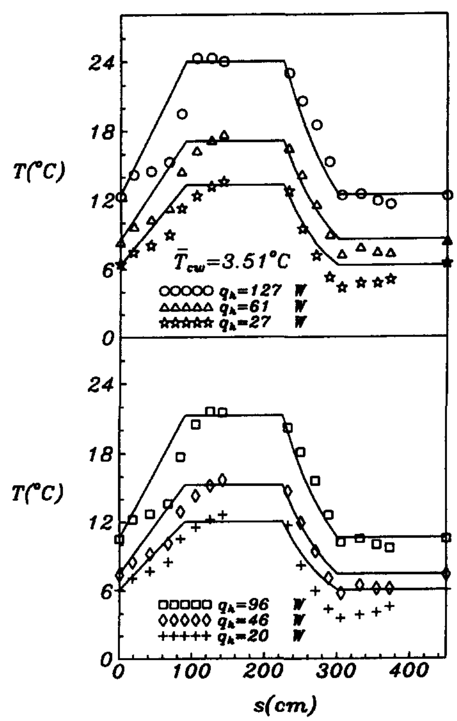

Fig. 4. Comparison of predicted and measured temperature distribution at various values of heat input $\bar{T}_{cw} = 3.51°\mathrm{C}$.

RESULTS AND DISCUSSION

A total of 40 different tests for the rectangular loop considered has been undertaken to explore the influence of density inversion of water with the pertinent parameters varying in the following ranges: the heat input through the heater section q_h = 20, 28, 46, 62, 96, 128 W; and the length-averaged temperature of the cooler section $\bar{T}_{cw}$ = 2.0, 3.0, 3.5, 4.6, 6.4, 8.4, 9.9°C.

First of all, Figs 3 and 4 exemplify comparison of the measured and predicted steady state fluid temperature distributions along the loop under various heat inputs and two different values of $\bar{T}_{cw}$. In conformity with equations (8a)–(8d) of the 1-D model, the fluid temperature is predicted to increase linearly along heater section and to decrease exponentially through the cooler section of the loop. At the given value of $\bar{T}_{cw}$, the fluid temperature profile shifts upward with the increase of heat input at the heater section. The predicted fluid temperature profiles appear to agree fairly well with the measured data.

There exists a marked discrepancy along the heater section particularly at the condition of high heat input, as shown in Figs 3 and 4. In contrast to the linear temperature increase predicted by the 1-D model, the measured fluid temperature can be seen to exhibit a somewhat nonlinear increase over the heater section, which becomes further distinct with the increase of heat input. This may be due to the experimental errors associated with measuring location of the thermocouples and heat exchange between the loop and the ambient as well as to the uncertainties of the fluid friction factor correlation and heat transfer coefficient used in the model. In addition, some of the idealizations adopted in the 1-D model, such as cross-sectional lumping of velocities and temperatures as well as neglect of heat conduction along the structural components, may contribute the disagreement.

Moreover, to illustrate the influence of density inversion of water on the thermal characteristics of the rectangular loop, the measured and predicted fluid temperature rise along the heater section, ΔT_h, are compared, as depicted in Fig. 5 which the measured and predicted values are displayed versus the heat input q_h for different values of $\bar{T}_{cw}$. The symbols denote the measured data and the solid lines the predictions from the present 1-D model. Also included in the figure are the predicted results, denoted by dashed lines based on the 1-D model adopting the linear density-temperature relation, namely not taking the density inversion effect into account. As expected, the temperature rise over the heater section exhibits a monotonic increase with the increase of the heating power. An overview of the figure reveals that the predictions based on the nonlinear density-temperature relation agree quite well with the measured results, a clear indication of significance of the density inversion effect on the performance of the circulating loop. Effect of the density inversion can be further inferred from the predictions from the model neglecting the density inversion based on equations (7) and (8), also shown in Fig. 5. The predicted values of ΔT_h appear to increasingly deviate from the measured data as well as from the predictions with density-inversion effect as the value of $\bar{T}_{cw}$ decreases from 9.8°C toward or below 4°C. It can, thus, be concluded that the thermal characteristics of rectangular water-filled natural circulation loop can have a significant bearing with occurrence of the density inversion phenomenon of water when the cooler section wall attains a temperature near 4°C.

Next the heat transfer results are presented in terms of nondimensional quantities of Nusselt and Rayleigh number, which are defined, respectively, as follows

$$Nu = \frac{q_h}{\pi k L_h(\bar{T}_{hw} - \bar{T}_{cw})} \tag{9}$$

and

$$Ra = \frac{g\beta(\bar{T}_{hw} - \bar{T}_{cw})D^3}{\nu\alpha} \tag{10}$$

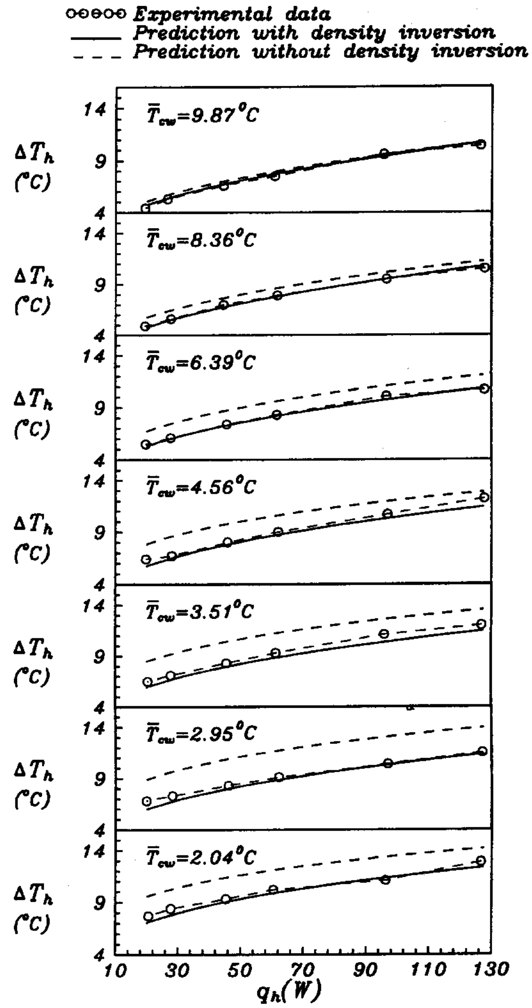

Fig. 5. Comparison of the predicted and measured temperature rise over the heated section of loop for various heating inputs and $\bar{T}_{cw}$.

The fluid properties were evaluated at the mean temperature between $\bar{T}_{hw}$ and $\bar{T}_{cw}$. Figure 6 illustrates a plot of the Nusselt number vs the Rayleigh number

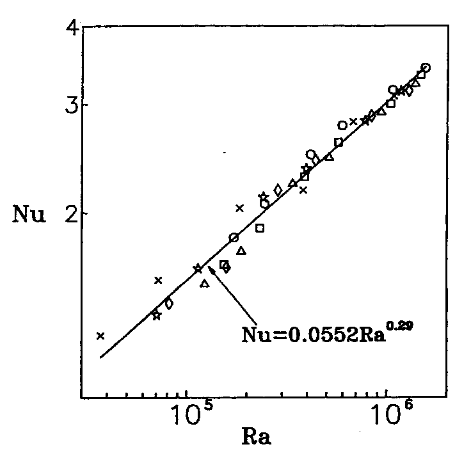

Fig. 6. Relation of Nusselt number with Rayleigh number.

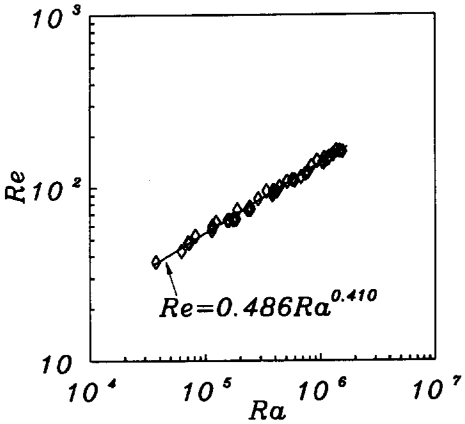

Fig. 7. Dependence of Reynolds number on Rayleigh number.

for the measured data. An empirical correlation developed on the basis of least squares fit of the data was of the form

$$Nu = 0.0552Ra^{0.29} \tag{11}$$

which has an averaged deviation of 3.42% with the measured data. From a closer examination of the correlation curve corresponding to equation (11), also shown in Fig. 6, it reveals that significant disparity between the correlation and measurement occurs mainly for $Ra \leqslant 2.5 \times 10^5$, which corresponds to the condition $\bar{T}_{cw}$ around 4°C. Moreover, the Nu–Ra correlation on the basis of the predicted values from the model with or without considering the density inversion effect was, respectively, determined to be of the following form

$$Nu = 0.0667Ra^{0.279} \text{ (with density–inversion effect)} \tag{12}$$

and

$$Nu = 0.119Ra^{0.251} \text{ (without density–inversion effect)} \tag{13}$$

The difference between these two theoretical correlations further demonstrates the significant role played by the density inversion effect on the heat transfer in a natural circulation loop. In addition, the great similarity between the correlation based on the predictions considering density–inversion, equation (12) and that of empirical correlation, equation (11), lends another validation for the present model in simulating thermal performance of a natural circulation loop with the density inversion effect.

Finally, Fig. 7 illustrates the power-law relationship between the Rayleigh number and the Reynolds number, $Re(=VD/\nu)$ based on the predicted cross-sectional averaged fluid velocity from the present model. Also plotted in Fig. 7 is the correlation curve obtained from a least-square regression of the predicted data, which is of the form

$$Re = 0.486Ra^{0.410} \tag{14}$$

which yields an averaged deviation of 2.65% from the predicted data.

CONCLUDING REMARKS

The results of a combined study of experiment and simulation for steady state heat transfer behavior of a rectangular natural circulation loop containing water near its density extreme have been presented. The 1-D theoretical model developed was found to be capable of predicting fairly well the steady state temperature distribution along the loop, in comparison with the experimental measurement. From the comparison between the simulations with or without considering the density inversion of water near 4°C, the influence of the density inversion on the heat transfer characteristics of the water-filled natural circulation loop has been clearly demonstrated to be substantially significant and needs to be taken into account when the cooler section attains a temperature around 4°C. Correlations for the Nusselt number as well as for the Reynolds number have been developed based on the predicted or measured results.

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